

# A-Level Physics 

## Angular Momentum

Mark Scheme

Time available: 78 minutes Marks available: 44 marks

## Mark schemes

1. (a) Attempt at calculating area above or below $t$ axis or both $\checkmark$
(Ang displacement =) $2.80+2.10-3.15=1.75 \mathrm{rad}$
$\left(\frac{1.75}{12.0}=\right) 0.15\left(\mathrm{rad} \mathrm{s}^{-1}\right) \checkmark$
Method must be valid
MP2: correct answer only
(calculator value $=0.145833$ )
MAX1 if counting square method used and answer rounds to 0.15 (rad s${ }^{-1}$ )
(b) $\quad P=T \omega$ giving $546(\mathrm{~W}) \checkmark$

Allow ecf for 590 (W) from
using $\omega_{1}=1.5 \mathrm{rad} \mathrm{s}^{-1}$
(c) Selects steepest part of graph and
determines gradient $\alpha=\frac{1.40--0.90}{5.0}=0.46\left(\mathrm{rad} \mathrm{s}^{-2}\right) \checkmark_{1}$
$T=I \alpha=9660 \mathrm{Nm} \mathfrak{V}_{2}$
Adds friction torque to give 10100 ( N m) $\sqrt{3}$
Accept any correct calculation of steepest graph slope: eg from $2 s$
to 5 s
$a=\frac{1.4}{3.0}=0.467$ giving $T=9800 \mathrm{~N} \mathrm{~m}$
or $5 s$ to $7 s$
$a=\frac{0.9}{2.0}=0.45$ giving $T=9450 \mathrm{Nm}$
Allow ECF from MP2 to MP3
Treat 10000 (Nm) as a 2 sf answer if consistent with their working.
(d) (net) $T \times t=9660 \times 5.0=4.8 \times 10^{4}(\mathrm{~N} \mathrm{~m} \mathrm{~s}) \checkmark$

OR
$\Delta(I \omega)=2.1 \times 10^{4}(1.40-(-0.90))=4.8 \times 10^{4}(\mathrm{~N} \mathrm{~m} \mathrm{~s}) \checkmark$
For first method allow ECF for torque $\checkmark_{2}$ from (c), but not for $\checkmark_{3}$ value
(calculator value $=48300$ )
(e)


Tick $(\checkmark)$ against 3rd box
2. (a) (The M of I decreases) because more mass closer to axis of rotation $\checkmark_{1}$ $I \omega$ /angular momentum constant since no external torque $\checkmark_{2}$ since $I$ decreases, $\omega$ must increase $\sqrt{3}$

For $\checkmark_{1}$ must have the idea of mass distribution around axis of rotation. Do not accept answers which give only decrease in radius as reason for lower $M$ of $I$.
For $\checkmark_{2}$ condone answers which do not mention the condition of no external torque. $\checkmark_{3}$ cannot be awarded if conservation of rotational kinetic energy used.
(b) $\quad I_{1} \omega_{1}=I_{2} \omega_{2} \quad \omega_{1}=4.3 \mathrm{rad} \mathrm{s}^{-1} \checkmark$

Accept the answer $4.3 \mathrm{rad} \mathrm{s}^{-1}$ if no working shown.
(c) Finds time for one rotation $\checkmark$

Divides 1.2 by time
AND
gives answer for complete rotations, not rounded up. $\checkmark$
time for 1 rotation $=2 \pi / 14.2=0.442 \mathrm{~s}$
1.2/0.442 = 2.7 rotations/turns/somersaults.

OR Angle turned through $=14.2 \times 1.2=17.04 \mathrm{rad}$
$17.04 / 2 \pi=2.7$ rotations
OR

Finds angle turned through in $1.2 \mathrm{~s} \checkmark$
Divides by $2 \pi$
AND
gives answer for complete rotations, not rounded up. $\checkmark$
Expect to see 2 complete
rotations/turns/somersaults.
For MP2 give CE for time or angle from MP1
(d) Any 2 from:

- build up a greater initial angular speed around the bar $\checkmark$ so reaches a greater height/will rotate faster in tuck $\checkmark$
- release at a greater angle from the horizontal $\checkmark$ so will rise to greater height giving more time for somersaults $\checkmark$
- get into tuck position earlier/get out of tuck position later $\checkmark$ so turning for more time $\checkmark$
- get into tighter tuck position $\checkmark$ reducing $I_{2}$, and increasing $\omega_{2} \checkmark$

Any 2
statement $\checkmark$ and correct reason $\checkmark$ scores 2 marks for each.
3. (a) The (total) angular momentum (of a system) remains constant provided no external torque acts (on the system) $\checkmark$

Must see 'angular'. Condone 'is conserved' for 'is constant'
Allow ang momtm before equals/is same as ang momtm after OR
initial ang momtm = final ang momtm
Allow I $\omega$ is constant if symbols explained
Do not allow 'force' in place of 'torque'
(b) Use of $I=I_{\mathrm{BODY}}+2 \times m r^{2} \checkmark$
$I_{1}=\left(71+2 \times 5.0 \times 4.1^{2}\right)=239 \mathrm{~kg} \mathrm{~m}^{2} \checkmark$
( $\approx 240 \mathrm{~kg} \mathrm{~m}^{2}$ )
For 2 marks 239 must be seen
(c) M of I decreases $\checkmark$

Because more mass closer to axis OR (for pods) $\underline{I=(\Sigma) m r^{2} \text { with } r \text { less } \checkmark ~}$
$I \omega$ / angular momentum remains constant/is conserved
(So as $I$ decreases) $\omega$ must increase $\checkmark$
Condone 'inertia' for 'moment of inertia'
2nd mark is for the reason why I is decreasing. Answer must relate to pods or masses getting closer to the axis. 'radius decreasing' on its own is not enough. Accept: pods get closer to axis/body as this implies mass is getting closer.
Both points needed for 3rd mark
(d) (Applies conservation of angular momentum $/ I_{1} \omega_{1}=I_{2} \omega_{2}$ )
$I_{1} \omega_{1}=240 \times 1.3=(312(\mathrm{~N} \mathrm{~m} \mathrm{~s})) \checkmark$
$312=\left(71+2 \times 5.0 \times 0.74^{2}\right) \omega_{2}$
$\omega_{2}=4.08 \mathrm{rad} \mathrm{s}^{-1} \checkmark$
Therefore max speed not reached OR arms can be retracted safely $\sqrt{ }$
OR
$I_{1} \omega_{1}=240 \times 1.3=(312(\mathrm{~N} \mathrm{~m} \mathrm{~s})) \checkmark$
$312=\left(71+2 \times 5.0 \times r_{2}{ }^{2}\right) 4.2$
$r_{2}=0.57 \mathrm{~m} \checkmark$
So with $r$ at circumference max speed not reached OR arms can be retracted safely $\checkmark$
OR
$I_{1} \omega_{1}=240 \times 1.3=(312(\mathrm{~N} \mathrm{~m} \mathrm{~s})) \checkmark$
$312=4.2 I_{2}$ at safety limit
$I_{2}=74(.3) \mathrm{kg} \mathrm{m}^{2} \checkmark$
Actual $I_{2}=76.5 \mathrm{~kg} \mathrm{~m}^{2}$
Therefore max speed not reached OR arms can be retracted safely $\sqrt{ }$
Using $239 \mathrm{~kg} \mathrm{~m}^{2}$ instead of $240 \mathrm{~kg} \mathrm{~m}^{2}$ leads to

$$
\omega^{2}=4.06 \mathrm{rad} \mathrm{~s}^{-1}
$$

Useful: $I_{2}=76.5 \mathrm{~kg} \mathrm{~m}^{2}$
Only credit last mark if conservation of angular momentum is used
Allow a judgement based on incorrect working (eg AE) provided conservation of angular momentum is used
Using $239 \mathrm{~kg} \mathrm{~m}^{2}$ instead of $240 \mathrm{~kg} \mathrm{~m}^{2}$ leads to
$r_{2}=0.55 \mathrm{~m}$
4. (a) The (total) angular momentum (of a system) remains constant provided no external torque acts (on the system) $\checkmark$

Do not accept 'force' in place of 'torque'
(b) $\quad I$ is the sum of the $m r^{2}$ products for point masses $m$ at radius $r \checkmark$

Or WTTE
Not $m$ is the mass and $r$ the radius - must refer to point or small masses or distribution of mass

OR
$\Sigma m r^{2}$ with $m$ and $r$ defined
OR
$I$ is a measure of the mass and the way the mass is distributed about an axis
(Small change in $r$ ) gives large change in $r^{2}$, hence large change in $/$
OR even though $m$ of panels is small, much of $m$ is at a greater radius and radius is squared $\checkmark$

$$
\text { For } 2^{\text {nd }} \text { mark must refer to effect of } r^{2} \text {. }
$$

(c) Angular momentum $=110 \times 5.2=572 \checkmark$
$\mathrm{Nm} \operatorname{siRkg~m} \mathrm{s}^{-1} \checkmark$
accept
$\mathrm{kg} \mathrm{m} \mathrm{m}^{2} \mathrm{rad} \mathrm{s}^{-1}$
(d) (Use of conservation of ang momtm) $572=230 \times \omega_{2} \checkmark$
$\omega_{2}=572 / 230=2.49 \mathrm{rad} \mathrm{s}^{-1} \checkmark$
5. (a) Law of conservation of angular momentum applies and $I_{1} \omega_{1}=I_{2} \omega_{2}$ OR Law of conservation of angular momentum applies and angular momentum $=I \omega \checkmark$ (because no external torque acts)

Adding plasticine increases $I \checkmark$
So $\omega$ must decrease to maintain I $\omega$ constant / to conserve angular momentum $\checkmark$
(b) $\quad I \times 3.46=\left(I+0.016 \times 0.125^{2}\right) \times 3.31 \checkmark$ $I=0.00552 \mathrm{~kg} \mathrm{~m}^{2} \checkmark 3 \mathrm{sf} \checkmark$

Useful: $m r^{2}=2.5 \times 10^{-4}$
Sig fig mark s an independent mark
If method correct but incorrect conversion of $g$ to kg or mm to m , award 1 mark out of first 2 marks
(c) (i) $\Delta \mathrm{E}=1 / 2 I \omega_{1}^{2}-1 / 2\left(I+m r^{2}\right) \omega_{2}{ }^{2}$

$$
=\left[1 / 2 \times 5.52 \times 10^{-3} \times 3.46^{2}\right]-
$$

$$
\left[1 / 2 \times 5.77 \times 10^{-3} \times 3.31^{2}\right] \checkmark
$$

$$
=1.39 \times 10^{-3} \mathrm{~J} \checkmark
$$

CE for I of turntable or I of plasticine from $2 b$
Answers will vary depending on rounding e.g. accept $1.43 \times 10^{-3}$
(ii) Work done against friction / deforming plasticine as it collides with turntable / to move or acclerate plasticine $\checkmark$

Allow heat loss on collision
Do not allow energy to sound

