A-Level Physics

Concept of Moment of Inertia

Mark Scheme

Time available: 54 minutes
Marks available: 43 marks

Mark schemes

1. (a) Attempt to use work done = force × distance with either incline work or resistance work or both ✓$_1$

Work done by flywheel

= [(1.46 × 10^4 × 9.81 × sin 5°) + 1.18 × 10^3] × 500 ✓$_2$

(= 6.83 × 10^6 J)

$1/2\ I\omega^2$= 6.83 × 10^6 giving ω = 468 (rad s^{-1}) ✓$_3$

MP1: award mark for valid attempt to calculate

mgh or $F \times s$ or both

mgh = 6.24 × 10^6 J

$F \times s$ = 5.9 × 10^5 J

MP2 for correct calculation of work done

MP3 for using their work done and $1/2\ I\omega^2$ to calculate ω

ECF for ✓$_3$

3

(b) ✓$_1$ for idea of use of flywheel as brake

✓$_2$ for idea of storing and reusing this energy

E_p change of tram can be converted to E_k of flywheel so less energy transferred to brakes/brakes last longer/tram will not reach a high speed ✓$_1$

OR

Energy otherwise dissipated/lost in brakes can be fed back to flywheel ✓$_1$

Fly wheel is charged/stores energy and energy can be used for later acceleration/driving ✓$_2$

OR

Fly wheel is charged/stores energy and at next stop less recharging energy will be needed. ✓$_2$

Give two marks if both points covered in their answer to part 1

Treat as neutral answers in terms of providing a smoother ride or less wear on parts due to connecting and reconnecting flywheel.

If no other marks are given, allow 1 MAX for a correct reference to regenerative braking.

2

(c) **The mark scheme gives some guidance as to what statements are expected to be seen in a 1 or 2 mark (L1), 3 or 4 mark (L2) and 5 or 6 mark (L3) answer.**
Guidance provided in section 3.10 of the 'Mark Scheme Instructions' document should be used to assist in marking this question.

Mark	Criteria
6	The factors which affect E_k and all three areas of shape, material and design for high ω will be covered in some detail. 6 marks can be awarded even if there is an error and/or if parts of one aspect are missing.
5	The factors which affect E_k and all three areas will be covered, at least two in detail.
4	Two areas successfully discussed, or one discussed and two others covered partially. Whilst there will be gaps, there should only be an occasional error.
3	One area discussed and one discussed partially, or all three covered partially. There are likely to be several errors and omissions in the discussion.
2	Only one area discussed or makes a partial attempt at two areas.
1	None of the three areas covered without significant error.
0	No relevant analysis.

examples of the points made in the response

- E_k proportional to ω^2
- E_k proportional to I
- for same mass of tram I or ω increased but not mass of flywheel

Shape

- I depends on mass and distribution of mass around axis
- ($I = \sum mr^2$ so) arrange more m at outer edge of flywheel
- by using heavy rim and spokes/thin centre web
- increase radius

Material

- use higher density material at rim
- use material of higher tensile strength / breaking stress
- for higher speeds without bursting/to withstand rotational/centripetal stresses
- eg titanium, CFRP

<u>Design for high ω</u> - increase ω by:

- reduce friction at bearings
- use lubrication or roller bearings/air bearings/magnetic bearings
- smooth outer surfaces / encase in vacuum
- small increase in ω gives large increase in E_k (because ω^2)

Also allow

- sketches which convey correct info clearly
- use of 'depends on' for 'proportional to'
- need for perfect balance
- gyroscopic effects

6

[11]

2. (a) Sum of all constituent masses × their radius/distance from the axis squared

Allow Σmr^2 with m defined as small mass or constituent mass or particle at a radius r and Σ explained.

Condone: 'from the axis' missing

Condone: a quantity expressing a body's tendency to resist angular acceleration/change in angular speed

1

(b) E_P lost (by falling mass) = E_K pulley + E_K mass ✓

$0.5Mgh = ½(0.5M)v^2 + ½(0.5MR^2)\,\omega^2$
Cancel 0.5 and M and substitute $\omega = {}^{v}/_{R}$ for ω
gives $gh = ½v^2 + ½v^2 = v^2$ ✓
use of $v^2 = u^2 + 2as$ giving $v^2 = 2ah$ ✓
substitutes $v^2 = 2ah$ in $gh = v^2$ (so $a = 0.5g$) ✓

*1st mark for equating E_P lost by mass to E_K gained by **both** mass and pulley. Accept this step in words or symbols*

2nd mark for $gh = v^2$

3rd mark for v^2 in terms of h

4th mark for combining correctly (to get $a = 0.5g$)

OR
$0.5Mg - F = 0.5Ma$ ✓
Torque = $I\alpha$ $F \times R = (0.5MR^2)\alpha$ ✓
(giving $F = 0.5\ MR\alpha$)
and substitute $\alpha = a/R$
leading to $F = 0.5Ma$ ✓
Substitute for F in $0.5Mg - F = 0.5Ma$ (gives $a = 0.5g$) ✓

OR with F or other letter as tension in string:

1st mark for Newton's 2nd law applied to mass in words or symbols

2nd mark for accelerating torque equation

3rd mark F in terms of a

4th mark for substituting to get $a = 0.5g$

Note: $\alpha = a/R$ is not in the spec, but students may know it and use this route.

Give ECF if M is used for the falling mass in place of $0.5M$

4

(c)

Route 1	Route 2
M of I spoked pulley is greater ✓$_1$ Reason given for greater M of I but must have reference to distribution or spread of mass about axis ✓$_2$ Greater proportion of E_P loss given to pulley OR lower prop to E_K of falling mass ✓$_3$ v of mass in same time will be lower so acceleration lower ✓$_4$	M of I spoked pulley is greater ✓$_1$ Reason given for greater M of I but must have reference to distribution or spread of mass about axis ✓$_2$ Presents valid argument relating I to α ✓$_3$ $\alpha = a/R$ (with α less) so acceleration of mass is less OR wheel turns through fewer rotations in same time so point on rim moves less distance so acceleration less ✓$_4$

WTTE

For ✓$_3$ and ✓$_4$ marks in route 2

$0.5M(g - a) = F$

$0.5M(g - a)R = I\,\alpha$

$0.5Mg = a\,(0.5M + I/R^2)\,a$

If I increases, a decreases.

Max 3

[7]

3. (a)

Translational dynamics	Rotational dynamics
force	torque ✓
mass	moment of inertia ✓

Do not allow 'inertia'

2

(b) $I_T = 2.6 \times 10^7 + (2.2 \times 10^3 \times 35^2) = 2.9 \times 10^7$ (kg m^2) ✓

Mark only awarded for arriving at correct answer to more than 1 sf.

1

(c) Use of (total) area under graph = (angular) displacement/distance ✓

$\omega_{max}((½ \times 30) + 20 + (½ \times 45)) = 4.7$

$\omega_{max} = 0.082$ (rad s-1) ✓

Alternative route is area of trapezium

$½\ \omega_{max}\ (20 + 95) = 4.7$

2

(d) moment of inertia of rotating jib + load increases as trolley moves outwards ✓

reference to $T = I\alpha$ with T constant, so α decreases ✓

decreased α means longer time to stop(than 95 s) ✓

3

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4. Marks awarded for this answer will be determined by the Quality of Written Communication (QWC) as well as the standard of the scientific response.

Level 3 (5-6 marks)

The information conveyed by the answer is clearly organized, logical and coherent, using appropriate specialist vocabulary correctly. The form and style of writing is appropriate to answer the question.

All three bullet points will be addressed. Answers will relate E_K to the factors that give high M of I, and means by which the angular speed can be increased, with sensible suggestions concerning the mechanism and / or reducing friction.

Level 2 (3-4 marks)

The information conveyed in the answer may be less well organized and not fully coherent. There is less use of specialist vocabulary or specialist vocabulary may be used or spelled incorrectly. The form and style of writing is less appropriate.

The candidate may not tackle all of the bullet points in the question fully, but should have a fairly good idea of the factors that affect energy storage. Candidates are likely to relate E_K to angular speed^2 and to the way the mass is distributed around the axis.

Level 1 (1-2 marks)

The information conveyed by the answer is poorly organised and may not be relevant or coherent. There is little correct use of specialist vocabulary.

The candidate shows little understanding of the factors which affect energy storage in a flywheel. They may relate rotational E_K to angular speed and / or M of I, but not confidently cover aspects of mass, and distribution of mass around the axis, and may not relate their answers well to the context of the question.

0 marks

The information conveyed by the answer is sketchy, and neither relevant or coherent.

The candidate shows inadequate understanding of the concept of moment of inertia. Formulae may be quoted from the Formulae booklet, but the candidate is unable to apply their meaning to the question.

examples of the points made in the response

- E_K proportional to ω^2
- E_K proportional to I

Shape

- I depends on mass and distribution of mass around axis.
- $I = \Sigma mr^2$ so arrange more m at outer edge of flywheel
- By using heavy rim and spokes / thin centre web
- Increase thickness (to increase m or I)

Material

- Higher density material
- Gives greater mass for given size

Mechanism

Increase ω by:

- changing gear ratio / reduce size of small gears / give higher ω per push
- Longer rack segment or more teeth on rack
- Reduce friction at bearings or between gears
- use lubrication or roller bearings
- small increase in ω gives large increase in E_K (because ω^2)

extra information

Also allow

- *use material of higher tensile strength*
- *for higher speeds without bursting / to withstand rotational stresses*
- *use magnet of greater mass*
- *answers in terms of lowering electromagnetic drag*
- *sketches which convey correct info clearly*
- *use of 'depends on' for 'proportional to'*

Level 3: 1st 2 points plus 6 other points from shape, material and mechanism

Level 2: between 4 and 7 points

Level 1 fewer than 4 points.

Quoting formulae alone is not enough.

Ignore references to strength of spring.

[6]

5.

(a) $\frac{3.5}{(2\pi \times 0.088)}$ = 6.3 rev

6.3 × 2π = 39.8 rad or 40 rad ✓

OR

$\frac{3.5}{0.088}$ = 39.8 or 40 rad ✓

If correct working shown with answer 40 rad give the mark
Accept alternative route using equations of motion

1

(b) $\omega = v / r = 2.2 / 0.088 = 25$ rad s^{-1} ✓

1

(c) (i) $E = ½I\omega^2 + ½mv^2 + mgh$
$= (0.5 \times 7.4 \times 25^2)$
$+ (0.5 \times 85 \times 2.2^2)$
$+ (85 \times 9.81 \times 3.5)$
= 2310 ✓
+ 206 ✓
+ 2920 ✓
(= 5440 J or 5400 J)

CE from 1b
$½ I \omega^2 + ½mv^2 = 2310 + 210 = 2520$ J
$½ I \omega^2 + mgh = 2310 + 2920 = 5230$ J
$½mv^2 + mgh = 210 + 2920 = 3130$ J
Each of these is worth 2 marks

3

(ii) Work done against friction = $T\theta$
= 5.2 × 40 = 210J ✓
Total work done = ***W*** = 5400 + 210
= 5600J ✓ 2 sig fig ✓

CE if used their answer to i rather than 5400J
Accept 5700 J (using 5440 J)
Sig fig mark is an independent mark

3

(d) Time of travel = distance / average speed = 3.5 / 1.1 = 3.2s ✓

$P_{ave} = \frac{5600}{3.2} = 1750$ W

$P_{max} = P_{ave} \times 2 = 3500$ W ✓

OR accelerating torque = $T = W / \theta$
= 5600 / 40 = 140 N m ✓
P = $T\, \omega_{max}$ = 140× 25 = 3500 W ✓

CE from ii

1780 W if 5650 J used

2

[10]