

# Gravitational Fields 

Mark Scheme

Time available: 77 minutes Marks available: 61 marks

1. (a) $\mathbf{G}$ has a greater mass than $\mathbf{H}$ with a reason $\checkmark_{1}$
because the null point is closer to $\mathbf{H} \checkmark_{2}$
$\checkmark_{1}$ The reason can be the second mark
OR
Because the density of the field is greater around $\mathbf{G}$
(b) The lines given tangential arrows that flow towards $\mathbf{G}$ and $\mathbf{H} . \checkmark$

1

1
(d) Sketch must pass through coordinates $(R, 0.40),(2 R, 0.10)$ and $(3 R, 0.044) \checkmark$ Must be within one small division of coordinates requested.

(e) The area underneath represents the energy/work needed (for an object) to move from $R$ to $2 R \checkmark$
of $1 \mathrm{~kg} /$ unit mass $\checkmark$
If no mark is scored then award a single mark for saying:
Area represents the gravitational potential difference between $R$ and $2 R$
(f) Use of $F=\frac{G M m}{r^{2}}$ to find the force between $\mathbf{P}$ and $\mathbf{H} \checkmark_{1}$

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\left(F_{(\mathrm{PH})}=1.8 \times 10^{13} \mathrm{~N}\right)
$$

(Calculation of the resultant force

$$
\begin{aligned}
& \left.F_{\text {total }}=\left(F_{(\mathrm{PH})}{ }^{2}+F_{(\mathrm{PG})^{2}}\right)^{1 / 2}\right) \\
& F_{\text {total }}=\left(\left\{1.8 \times 10^{13}\right\}^{2}+\left\{6.4 \times 10^{12}\right\}^{2}\right)^{1 / 2} \checkmark_{2} \\
& F_{\text {total }}=1.9 \times 10^{13} \checkmark_{3} \\
& \text { Use of } a=\frac{F}{m}=\frac{1.9 \times 10^{13}}{2.0 \times 10^{20}}=9.5 \times 10^{-8}\left(\mathrm{~m} \mathrm{~s}^{-2}\right) \checkmark_{4} \\
& \quad \quad \checkmark_{1} F_{(\mathrm{PH})}=\frac{6.67 \times 10^{-11} \times 3.0 \times 10^{25} \times 2.0 \times 10^{20}}{\left(1.5 \times 10^{11}\right)^{2}}
\end{aligned}
$$

$\checkmark_{2}$ Mark is for the use of the equation allowing for ecf from candidate's force calculation.
$\sqrt{ }$ Correct answer only, no ecf $\checkmark 4$ Allow ecf from $F_{\text {total }}$
(g) The resultant force is not (centripetal and) directed towards the centre of $\mathbf{H}$.
OR
A circular orbit does not follow a gravitational equipotential (owtte)
$\checkmark$
The answer can focus on the conditions necessary for circular motion eg the need for a centripetal force.
Or
At different locations on a circular path the total gravitational potential energy is different which requires energy which is not provided.
2. (a) (centripetal) force $=m r(2 \pi / T)^{2} \operatorname{Ormr}(\omega)^{2}$
(is given by the gravitational) force $=G m M / r^{2} \checkmark$ (mark for both equations)
(equating both expressions and substituting for $\omega$ if required) $T^{2}=$ ( $4 \pi^{2} / G M$ ) $r^{3} \checkmark\left(4 \pi^{2} / G M\right.$ is constant, the constants may be on either side of equation but $T$ and $r$ must be numerators)

First mark is for two equations (gravitational and centripetal)
The second mark is for combining.
(b) (use of $T^{2} \propto r^{3}$ so $\left.\left(T_{P} / T_{E}\right)^{2}=\left(r_{P} / r_{E}\right)^{3}\right)$
$\left(T_{P} / 1.00\right)^{2}=\left(5.91 \times 10^{9} / 1.50 \times 10^{8}\right)^{3} \checkmark$ (mark is for substitution of given data into any equation that corresponds to the proportional equation given above)
$\left(T_{P}{ }^{2}=61163\right)$
$T_{P}=250(\mathrm{yr}) \checkmark(247 \mathrm{yr})$
Answer only gains both marks
The calculation may be performed using data for the Sun in $T^{2}=$ $\left(4 \pi^{2} /\right.$ GM) $r^{3}$ easily spotted from $M_{s}=1.99 \times 10^{30} \mathrm{~kg}$ giving a similar answer 247-252 yr.
(c) using $M\left(=g r^{2} / G\right)=0.617 \times\left(1.19 \times 10^{6}\right)^{2} / 6.67 \times 10^{-11} \checkmark$
$\mathrm{M}=1.31 \times 10^{22} \mathrm{~kg} \checkmark$
answer to 3 sig fig $\checkmark$ (this mark stands alone)
The last mark may be given from an incorrect calculation but not lone wrong answer.
(d) Initial KE $=1 / 2(\mathrm{~m}) 1400^{2}=9.8 \times 10^{5}(\mathrm{~m}) \mathrm{J} \checkmark$

Energy needed to escape $=7.4 \times 10^{5}(\mathrm{~m}) \mathrm{J} \checkmark$
So sufficient energy to escape. $\checkmark$
OR For object on surface escape speed given by $7.4 \times 10^{5}=1 / 2 v^{2}$ $\checkmark$
escape speed $=1200 \mathrm{~m} \mathrm{~s}^{-1} \checkmark$ (if correct equation is shown the previous mark is awarded without substitution)
So sufficient (initial) speed to escape. $\checkmark$
OR escape velocity $=\sqrt{\frac{2 G M}{R}}$ substituting $M$ from part (c) $\checkmark$
escape speed $=1200 \mathrm{~m} \mathrm{~s}^{-1} \checkmark\left(1210 \mathrm{~m} \mathrm{~s}^{-1}\right)$
So sufficient (initial) speed to escape. $\checkmark$
OR escape velocity $=\sqrt{2 R g}$ substituting from data in (c) $\checkmark$
Third alternative may come from a CE from (c)
$\left(1.06 \times 10^{-8} \times\left(1.06 \times 10^{-8} \times \sqrt{\text { answer(c) })}\right)\right.$
Conclusion must be explicit for third mark and cannot be awarded from a CE
3. (a) the work done per unit mass $\checkmark$
in moving from infinity to the point $\checkmark$
(b) Gravitational potential is defined as zero at $\infty \checkmark$
(Forces attractive) so work must be done (on a mass) to reach $\infty$ (hence negative) $\checkmark$
(c) $V=-G M / r=6.67 \times 10^{-11} \times 5.9710^{24} / 6.37 \times 10^{6} \checkmark$
$=-6.25 \times 10^{7} \mathrm{~J} \mathrm{~kg}^{-1} \checkmark$
(d) in the plane of the equator
always above the same location on the earth
having the same period as the earth / 24 hours
$\checkmark \checkmark$ any two lines
(e) $V=-G M / r=6.67 \times 10^{-11} \times 5.97 \times 10^{24} / 4.23 \times 10^{7}=-9.41 \times 10^{6} \mathrm{Jkg}^{-1} \checkmark$
$\mathrm{E}_{\mathrm{p}}=\Delta \mathrm{V} \times m==(6.26-0.94) \times 10^{7} \times 1200 \checkmark$
$=6.38 \times 10^{10} \mathrm{~J} \checkmark$
(f) radius must increase $\checkmark$
velocity gets smaller $\checkmark$
reference to $R^{3}$ is proportional to $T^{2} \checkmark$
reference (from circular motion) $v^{2}$ is proportional to $1 / r \checkmark$
4. (a) Idea that both astronaut and vehicle are travelling at same (orbital) speed or have the same
(centripetal) acceleration / are in freefall Not falling at the same speed

No (normal) reaction (between astronaut and vehicle)
(b) (i) Equates centripetal force with gravitational force using appropriate formulae
E.g. $\frac{G M m}{r^{2}}=\frac{m v^{2}}{r}$ or $m r \omega^{2}$

Correct substitution seen e.g. $v^{2}=\frac{6.67 \times 10^{-11} \times 5.97 \times 10^{24}}{\text { anyvalue of radius }}$
B1
(Radius of) $7.28 \times 10^{6}$ seen or $6.38 \times 10^{6}+0.9 \times 10^{6}$
B1
$7396\left(\mathrm{~m} \mathrm{~s}^{-1}\right)$ to at least 4 sf
Or $v^{2}=5.47 \times 10^{7}$ seen
B1
(ii) $\quad \triangle \mathrm{PE}=6.67 \times 10^{-11} \times 5.97 \times 10^{24} \times 1.68 \times 10^{4}(1 /(7.28 \times$ $\left.\left.10^{6}\right)-1 /\left(6.78 \times 10^{6}\right)\right)$

C1
$-6.8 \times 10^{10} \mathrm{~J}$
C1
$\Delta K E=0.5 \times 1.68 \times 10^{4} \times\left(7700^{2}-7400^{2}\right)=3.81 \times 10^{10} \mathrm{~J}$
C1
$\Delta K E-\Delta P E=(-) 2.99 \times 10^{10}(\mathrm{~J})$

OR
Total energy in original orbit shown to be (-)GMm / $2 r$ or $m v^{2} / 2-G M m / r$

C1
Initial energy
$=-6.67 \times 10^{-11} \times 5.97 \times 10^{24} \times 1.68 \times 10^{4} /\left(2 \times 7.28 \times 10^{6}\right)$ $=4.59 \times 10^{11}$

C1
Final energy
$=-6.67 \times 10^{-11} \times 5.97 \times 10^{24} \times 1.68 \times 10^{4} /\left(2 \times 6.78 \times 10^{6}\right)$ $=4.93 \times 10^{11}$
$3.4 \times 10^{10}(\mathrm{~J})$
Condone power of 10 error for $C$ marks
5. (a) (i) Use of $F-G M m / r^{2}$

C1
Allow 1 for -correct formula quoted but forgetting square in substitution

Correct substitution of data
-missing m in substitution
491 (490)N
A1
-substutution with incorrect powers of 10 Condone 492 N,
(ii) Up and down vectors shown (arrows at end) with labels
allow $\mathrm{W}, \mathrm{mg}$ (not gravity); $\quad R$
allow if slightly out of line / two vectors shown at feet
up and down arrows of equal lengths
B1
condone if colinear but not shown acting on body
In relation to surface $W \leq R$ (by eye) to allow for weight vector starting in middle of the body
Must be colinear unless two arrows shown in which case $R$ vectors $1 / 2$ W vector(by eye)
(b) (i) Speed $=2 \pi r / T$

B1
Max 2 if not easy to follow
$2 \pi 6370000 /(24 \times 60 \times 60)$
B1
$463 \mathrm{~m} \mathrm{~s}^{-1}$
B1
Must be 3sf or more
(ii) Use of $F=m v^{2} / r$

A1
(iii) Correct direction shown (Perpendicular to and toward the axis of rotation) NB - not towards the centre of the earth

B1
(c) Force on scales decreases / apparent weight decreases

Appreciates scale reading $=$ reaction force

## C1

The reading would become 489 (489.3)N or reduced by 1.7 N )
A1
Some of the gravitational force provides the necessary centripetal force

$$
\text { or } R=m g-m v^{2} / r
$$

