

# Rotational Kinetic Energy 

Question Paper

Time available: 66 minutes Marks available: $\mathbf{3 6}$ marks

1. Figure 1 shows a yo-yo made of two discs separated by a cylindrical axle. Thin string is wrapped tightly around the axle.

Figure 1


Initially both the free end $\mathbf{A}$ of the string and the yo-yo are held stationary.
With A remaining stationary, the yo-yo is now released so that it falls vertically. As the yo-yo falls, the string unwinds from the axle so that the yo-yo spins about its centre of mass.

The linear velocity $v$ of the centre of mass of the falling yo-yo is related to the angular velocity $\omega$ by $v=r \omega$ where $r$ is the radius of the axle.
(a) The yo-yo accelerates uniformly as it falls from rest. The string remains taut and has negligible thickness.

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mass of yo-yo \(=9.2 \times 10^{-2} \mathrm{~kg}\)
radius of axle \(=5.0 \times 10^{-3} \mathrm{~m}\)
moment of inertia of yo-yo about axis \(\mathbf{X}-\mathbf{X}=8.6 \times 10^{-5} \mathrm{~kg} \mathrm{~m}^{2}\)
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When the yo-yo has fallen a distance of 0.50 m , its linear velocity is $V$.
Calculate $V$ by considering the energy transfers that occur during the fall.

$$
V=\ldots \mathrm{m} \mathrm{~s}^{-1}
$$

(b) The yo-yo falls further until all the string is unwound. The yo-yo then 'sleeps'. This means the yo-yo continues to rotate in a loose loop of string as shown in Figure 2.

Figure 2


The string applies a constant frictional torque of $8.3 \times 10^{-4} \mathrm{~N} \mathrm{~m}$ to the axle. The angular velocity of the yo-yo at the start of the sleep is $145 \mathrm{rad} \mathrm{s}^{-1}$.

Determine, in rad, the total angle turned through by the yo-yo during the first 10 s of sleeping.
angle =
$\qquad$ rad
2. The fly-press shown below is used by a jeweller to punch shapes out of a thin metal sheet.


The frame holds a screw and punch. Two arms are attached to the screw, each loaded with a heavy steel ball. The screw is driven downwards when the arms are rotated.
Kinetic energy is stored in the rotating parts: the balls, arms, screw and punch. This energy is used to punch the shape out of the metal sheet.
(a) When the punch reaches the metal sheet, the rotational speed of the arms is $2.9 \mathrm{rev} \mathrm{s}^{-1}$. At this speed the rotational kinetic energy of the rotating parts is 10.3 J .

Calculate the moment of inertia of the rotating parts about the axis of rotation.

$\qquad$ $\mathrm{kg} \mathrm{m}^{2}$
(b) The total mass of the screw, punch and arms is the same as the total mass of the two balls.

Explain why the moment of inertia of the screw, punch and arms about the axis of rotation is much smaller than the moment of inertia of the steel balls about the same axis.
$\qquad$
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$\qquad$
(c) During the punching of the metal sheet, the rotating parts of the fly-press are brought uniformly to rest from an initial rotational speed of $2.9 \mathrm{rev} \mathrm{s}^{-1}$ in a time of 89 ms .

## Determine

- the angular deceleration
- the angle turned through by the rotating parts.
angular deceleration $=$ $\qquad$ $\mathrm{rad} \mathrm{s}^{-2}$
angle $=$ $\qquad$ rad
(d) For thicker or stiffer metal sheets the rotational kinetic energy at $2.9 \mathrm{rev} \mathrm{s}^{-1}$ is not enough to punch out the shape.

The distance from the axis of rotation to the centre of each ball is $y$.
The radius of each ball is $R$.
The stored energy can be increased by
either

- increasing $y$ by $15 \%$ without changing $R$
or
- increasing $R$ by $15 \%$ without changing $y$.

Deduce which of these would produce the greater increase in stored energy.
$\qquad$
$\qquad$
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$\qquad$
$\qquad$
$\qquad$
(e) Which of the following is the SI unit for angular impulse?

Tick ( $\checkmark$ ) one box.
$\mathrm{Nm} \mathrm{s}^{-1}$


Ns


Nms

$\mathrm{kg} \mathrm{m} \mathrm{m}^{2} \mathrm{~s}^{-2}$

3. (a) A metal flywheel is rotating on frictionless bearings. The temperature is increased so that the flywheel expands.

Consider each of the following statements and indicate with a tick $(\checkmark)$ if it is correct.

|  | $\checkmark$ if correct |
| :--- | :--- |
| The moment of inertia will decrease. |  |
| The angular velocity will decrease. |  |
| The angular momentum will be unaltered. |  |

An electric motor drives a machine which stamps out shapes from sheet steel. The machine is fitted with a flywheel of moment of inertia $25 \mathrm{~kg} \mathrm{~m}^{2}$ which is accelerated uniformly until it is rotating at $640 \mathrm{rev} \mathrm{min}^{-1}$. The machine then starts a stamping operation which reduces the flywheel's angular speed to $360 \mathrm{rev} \mathrm{min}^{-1}$.
(b) Explain why a flywheel is fitted between the motor and the stamping machine.
$\qquad$
$\qquad$
$\qquad$
$\qquad$
$\qquad$
$\qquad$
$\qquad$
(c) Calculate the energy needed for the stamping operation.

> energy
$\qquad$ J
(d) Immediately after the stamping operation the flywheel is accelerated to its initial speed of $640 \mathrm{rev} \mathrm{min}^{-1}$ in a time of 5.0 s . The next stamping operation then begins.

Calculate the constant torque provided by the motor during this 5.0 s . Assume that the bearing frictional torque is negligible.

$$
\text { torque }=\ldots \mathrm{N} \mathrm{~m}
$$

(e) Calculate the minimum power output of the electric motor required.
power $\qquad$ W
(f) The flywheel is a solid disc. It is to be replaced with a flywheel which gives a smaller angular speed change for each stamping operation.

Two replacement flywheels, $\mathbf{A}$ and $\mathbf{B}$, are available and information about them and the original flywheel is given in the table below.

| flywheel | density of <br> material $/ \mathbf{k g ~ m}^{\mathbf{- 3}}$ | thickness of disc / $\mathbf{m}$ | outer radius / m |
| :---: | :---: | :---: | :---: |
| original | 7800 | 0.10 | 0.38 |
| A | 8800 | 0.20 | 0.30 |
| B | 2900 | 0.10 | 0.50 |

Deduce which flywheel, A or B, would be more suitable. Explain your choice.
The moment of inertia $I$ of a solid disc of mass $m$ and outer radius $r$ about an axis through the centre is given by

$$
I=\frac{1}{2} m r^{2}
$$

$\qquad$
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4. The figure below shows an experiment to determine the moment of inertia of a bicycle wheel. One end of a length of strong thread is attached to the tyre. The thread is wrapped around the wheel and a 0.200 kg mass is attached to the free end. The wheel is held so that the mass is at a height of 1.50 m above the floor. The wheel is released and the time taken for the mass to reach the floor is measured.

(a) State the energy transfers that take place from the moment the wheel is released until the mass hits the floor.
$\qquad$
$\qquad$
$\qquad$
$\qquad$
$\qquad$
$\qquad$
(b) Calculations based on the measurements made show that at the instant the mass hits the floor:

- the speed of the mass is $2.22 \mathrm{~m} \mathrm{~s}^{-1}$
- the wheel is rotating at $6.73 \mathrm{rad} \mathrm{s}^{-1}$
- the wheel has turned through an angle of 4.55 rad from the point of release.

A separate experiment shows that a constant frictional torque of $7.50 \times 10^{-3} \mathrm{~N} \mathrm{~m}$ acts on the wheel when it is rotating.

By considering the energy changes in the system, show that the moment of inertia of the wheel about its axis is approximately $0.1 \mathrm{~kg} \mathrm{~m}^{2}$.
(c) When the mass hits the floor the thread is released from the wheel.

Calculate the angle turned through by the wheel before it comes to rest after the thread is released.
angle $=$ $\qquad$ rad

