AQA

GCSE

MATHEMATICS

8300/1H

Higher Tier Paper 1 Non-Calculator

Mark scheme

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Version: 1.0 Final

Mark schemes are prepared by the Lead Assessment Writer and considered, together with the relevant questions, by a panel of subject teachers. This mark scheme includes any amendments made at the standardisation events which all associates participate in and is the scheme which was used by them in this examination. The standardisation process ensures that the mark scheme covers the students' responses to questions and that every associate understands and applies it in the same correct way. As preparation for standardisation each associate analyses a number of students' scripts. Alternative answers not already covered by the mark scheme are discussed and legislated for. If, after the standardisation process, associates encounter unusual answers which have not been raised they are required to refer these to the Lead Assessment Writer.

It must be stressed that a mark scheme is a working document, in many cases further developed and expanded on the basis of students' reactions to a particular paper. Assumptions about future mark schemes on the basis of one year's document should be avoided; whilst the guiding principles of assessment remain constant, details will change, depending on the content of a particular examination paper.

Further copies of this mark scheme are available from aqa.org.uk

Glossary for Mark Schemes

GCSE examinations are marked in such a way as to award positive achievement wherever possible. Thus, for GCSE Mathematics papers, marks are awarded under various categories.

If a student uses a method which is not explicitly covered by the mark scheme the same principles of marking should be applied. Credit should be given to any valid methods. Examiners should seek advice from their senior examiner if in any doubt.

M	Method marks are awarded for a correct method which could lead to a correct answer.
A	Accuracy marks are awarded when following on from a correct method. It is not necessary to always see the method. This can be implied.
B	Marks awarded independent of method.
ft	Follow through marks. Marks awarded for correct working following a mistake in an earlier step.
SC	Special case. Marks awarded for a common misinterpretation which has some mathematical worth.
M dep	A method mark dependent on a previous method mark being awarded.
B dep	A mark that can only be awarded if a previous independent mark has been awarded.
oe	Or equivalent. Accept answers that are equivalent. eg accept 0.5 as well as $\frac{1}{2}$
[a, b]	Accept values between a and b inclusive.
[a, b)	Accept values $a \leq$ value $< b$
3.14 …	Accept answers which begin 3.14 eg 3.14, 3.142, 3.1416
Use of brackets	It is not necessary to see the bracketed work to award the marks.

Examiners should consistently apply the following principles

Diagrams

Diagrams that have working on them should be treated like normal responses. If a diagram has been written on but the correct response is within the answer space, the work within the answer space should be marked. Working on diagrams that contradicts work within the answer space is not to be considered as choice but as working, and is not, therefore, penalised.

Responses which appear to come from incorrect methods

Whenever there is doubt as to whether a student has used an incorrect method to obtain an answer, as a general principle, the benefit of doubt must be given to the student. In cases where there is no doubt that the answer has come from incorrect working then the student should be penalised.

Questions which ask students to show working

Instructions on marking will be given but usually marks are not awarded to students who show no working.

Questions which do not ask students to show working

As a general principle, a correct response is awarded full marks.

Misread or miscopy

Students often copy values from a question incorrectly. If the examiner thinks that the student has made a genuine misread, then only the accuracy marks (A or B marks), up to a maximum of 2 marks are penalised. The method marks can still be awarded.

Further work

Once the correct answer has been seen, further working may be ignored unless it goes on to contradict the correct answer.

Choice

When a choice of answers and/or methods is given, mark each attempt. If both methods are valid then M marks can be awarded but any incorrect answer or method would result in marks being lost.

Work not replaced

Erased or crossed out work that is still legible should be marked.

Work replaced

Erased or crossed out work that has been replaced is not awarded marks.

Premature approximation

Rounding off too early can lead to inaccuracy in the final answer. This should be penalised by 1 mark unless instructed otherwise.

Continental notation

Accept a comma used instead of a decimal point (for example, in measurements or currency), provided that it is clear to the examiner that the student intended it to be a decimal point.

Question	Answer	Mark	Comments
1	9	B1	
2	$2\frac{7}{9}$	B1	
3	6π	B1	
4	$\frac{37}{8}$	B1	
5(a)	9.7×10^{-4}	B1	
	Additional Guidance		
	Condone $9.7 . 10^{-4}$ or $9.7 \cdot 10^{-4}$		B1
	Ignore zeroes before the '9' eg 00009.7×10^{-4}		B1
	$9.7 \times 10^{4-}$		B0

Question	Answer	Mark	Comments
5(b)	300 000 and 4000 or $(10^5 \div 10^3 =) 10^2$ or $(10^5 \div 10^3 =) 100$ or $7.5 \times 10^{(1)}$ or 75×10^0 or $\frac{3\times10^2}{4}$ or $\frac{300}{4}$	M1	
	75	A1	
	Additional Guidance		
	If the answer is given in standard form and as 75 the student must indicate that 75 is their chosen answer or it must be the final answer given eg1 $7.5 \times 10^{(1)} = 75$ on the answer line eg2 $75 = 7.5 \times 10^{(1)}$ on the answer line		M1A1 M1A0
	$\frac{300}{4}$ or 75 from incorrect working scores zero eg1 $3 \times 10^5 = 30\,000$ and $4 \times 10^3 = 400$ and $30\,000 \div 400 = \frac{300}{4} = 75$ eg2 $\frac{30\,000}{400} = 75$		M0A0 M0A0
	For the method mark, ignore incorrect work from a correct expression eg $0.75 \times 10^2 = 7.5 \times 10^3$		M1A0
	If the student attempts two methods (simplifying the powers and attempting to convert to ordinary numbers) mark both methods and award the higher mark		

Question	Answer	Mark	Comments
6(a)	$\frac{1}{6}$ on '1' and $\frac{1}{3}$ or $\frac{2}{6}$ on '2 or 3' and $\frac{1}{2}$ on each of 'Odd' and 'Even'	B2	oe fraction, decimal or percentage B1 $\frac{1}{6}$ on '1' and $\frac{1}{3}$ or $\frac{2}{6}$ on '2 or 3' or $\frac{1}{2}$ on each of 'Odd' and 'Even' or all correct unsimplified probabilities with one or more simplification errors eg $\frac{3}{6}$ on 'Odd' simplified to $\frac{1}{3}$
	Additional Guidance		
	Accept decimals or percentages rounded or truncated correctly to at least 2 significant figures		
	Only withhold a mark for simplification errors if B2 would otherwise be awarded		
	Ignore extra branches added		
	Ignore attempts to work out combined probabilities to the right of the tree diagram		
	If an answer line is blank, the student may have written their answer elsewhere on the branch		

Question	Answer	Mark	Comments
6(b)	**Alternative method 1: P(1) + P(4, 5 or 6) × P(Odd)**		
	$\frac{1}{2}$ × their $\frac{1}{2}$ or $\frac{1}{4}$	M1	oe
	their $\frac{1}{4}$ + their $\frac{1}{6}$	M1dep	oe
	(P(win) =) $\frac{10}{24}$ or $\frac{5}{12}$	A1ft	oe ft their tree diagram
	Lose (and P(Lose) = $\frac{14}{24}$ or $\frac{7}{12}$ oe)	A1ft	ft correct decision for their $\frac{5}{12}$ (and their $\frac{7}{12}$) with M2 scored
	Alternative method 2: 1 – P(2 or 3) – P(4, 5 or 6) × P(Even)		
	$\frac{1}{2}$ × their $\frac{1}{2}$ or $\frac{1}{4}$	M1	oe
	their $\frac{1}{4}$ + their $\frac{1}{3}$ or P(lose) = $\frac{7}{12}$	M1dep	oe ft their tree diagram
	(P(win) =) $\frac{10}{24}$ or $\frac{5}{12}$	A1ft	oe ft their tree diagram
	Lose (and P(Lose) = $\frac{14}{24}$ or $\frac{7}{12}$ oe)	A1ft	ft correct decision for their $\frac{5}{12}$ (and their $\frac{7}{12}$) with M2 scored
	Additional Guidance is on the following page		

Question	Answer	Mark	Comments

	Additional Guidance	
6(b) cont	Check the tree diagram for working	
	Any 'their' or ft probability must be > 0 and < 1 for marks to be awarded	
	For the second A1ft, the ft can be from an incorrect tree (which may score 4 marks) or an arithmetic error (which scores 3 marks, M1M1A0A1ft)	
	Accept equivalent fractions or decimals within calculations and equivalent fractions, decimals or percentages for final probabilities	
	Accept decimals or percentages rounded or truncated correctly to at least 2 significant figures	
	Condone $\frac{1}{2} \times$ their $\frac{1}{2}$ as part of a longer, incorrect multiplication eg $\frac{1}{2} \times \frac{1}{2} \times \frac{1}{6}$	M1M0A0A0
	Condone decimals used within fractions eg P(Win) = $\frac{2.5}{6}$	at least M1M1A1
	For the method marks, condone incorrect mathematical notation eg $\frac{1}{2} \times \frac{1}{2} = \frac{1}{4} + \frac{1}{6} = \ldots$	at least M1M1 (may go on to score 3 or 4 marks)
	For the second A1ft, if the student gives a value for P(Lose), their P(Win) + their P(Lose) must equal 1 However, allow a comparison to $\frac{1}{2}$ unless it is clearly an incorrect value for P(Lose)	

Question	Answer	Mark	Comments
7	**Alternative method 1**		
	$3 \div \frac{20}{100}$ or 3×5 or 15 or 3×6	M1	oe
	18	A1	
	Alternative method 2		
	$1.2x = x + 3$	M1	oe equation
	18	A1	
	Additional Guidance		
	Trial and improvement scores 0 or 2 unless M1 can be awarded for 15		
	15 seen scores M1		

Question	Answer	Mark	Comments
8	$(3^{12} =)\ 531\,441$ or $(3^5 =)\ 243$ or $(3^{12} \div 3^5 =)\ 3^7$ or $(3^{12} \div 3^5 =)\ 2187$ or $(3^2 \times 3 =)\ 3^3$ or $(3^2 \times 3 =)\ 27$ or $3^{12} \div 3^5 \div 3^2 \div 3$ or $\frac{3^{12}}{3^5} \times \frac{1}{3^2 \times 3}$	M1	
	$3^7 \div 3^3$ or $3^7 \div 27$ or $3^{(12-5-2-1)}$ or $\frac{3^{12}}{3^8}$ or 3^4 or $2187 \div 27$	M1dep	oe in the form $3^{n} \div 3^{(n-4)}$
	81	A1	
	Additional Guidance		
	3^4 and 81 on the answer line in either order		M1M1A1
	81 in working and 3^4 on the answer line		M1M1A0

Question	Answer	Mark	Comments
9	**Alternative method 1: areas**		
	$\pi \times 10^2$ or 100π	M1	implied by [314, 314.2]
	$\pi \times (8 \div 2)^2$ or $\pi \times 4^2$ or 16π or $\pi \times (8 \div 2)^2 \div 2$ or $\pi \times 4^2 \div 2$ or $16\pi \div 2$ or 8π	M1	implied by [50.2, 50.3] or [25.12, 25.14] 92π or 84π or 92 : 8 or 8 : 92 or 84 : 16 or 16 : 84 implies M1M1
	(their $100(\pi)$ – their $8(\pi)$) ÷ their $8(\pi)$ or $92(\pi) \div 8(\pi)$ or their $100(\pi) \div$ their $8(\pi)$ (– 1) or $12\frac{1}{2}$ (– 1) or 12.5 (– 1)	M1dep	dep on M2 absence of π must be consistent condone $16(\pi)$ as their $8(\pi)$ in first calculation only, ie condone (their $100(\pi)$ – their $16(\pi)$) ÷ their $16(\pi)$ or $84(\pi) \div 16(\pi)$, but not their $100(\pi) \div$ their $16(\pi)$ (– 1)
	$11\frac{1}{2}$ or 11.5	A1	condone $\frac{23}{2}$
	Alternative method 2: scale factor		
	$\frac{10}{8 \div 2}$ or $\frac{10}{4}$ or $\frac{5}{2}$ or $\frac{10 \times 2}{8}$ or $\frac{20}{8}$ or 2.5	M1	oe scale factor of lengths eg $\frac{2}{5}$ or 0.4 accept 2 : 5 or 5 : 2 oe ratio π may be present, but must be consistent in numerator and denominator
	(their $\frac{5}{2}$)^2 or $\frac{25}{4}$	M1dep	oe scale factor of areas eg $\frac{4}{25}$ accept 4 : 25 or 25 : 4 oe ratio
	2 × their $\frac{25}{4}$ (– 1) or $\frac{25}{2}$ (– 1) or $12\frac{1}{2}$ (– 1) or 12.5 (– 1)	M1dep	oe eg 2 ÷ their $\frac{4}{25}$ (– 1)
	$11\frac{1}{2}$ or 11.5	A1	condone $\frac{23}{2}$
	Additional Guidance is on the following page		

Question	Answer	Mark	Comments

	Additional Guidance	
9 (cont)	Accept, for example, $\pi 8$ or $\pi \times 8$ or $8 \times \pi$ for 8π	
	An answer of 11.5π with no incorrect working	M1M1M1A0
	Consistent use of πd^2 for the area of a circle gives the area of the circle as 400π, the area of the semicircle as 32π and the area of the shaded part as 368π. This also gives the answer 11.5, but scores zero	M0M0M0A0
	Irrespective of where their answer comes from and the presence of other measures such as circumference, students can gain the first two marks of alternative method 1 if it is clear that the methods or values given are for area eg 1 Big area = 100π, little area = 8π, big circumference = 20π, little circumference = 4π, $20 \div 4 = 5$ eg 2 100π, 8π, 20π, 4π	M1M1M0A0 M0M0
	Do not award the second mark if the value of 8π comes from πd This is implied by, eg, 'Area of circle = 20π, area of semi-circle = 8π'	M?M0 M0M0
	$\frac{100(\pi) - 16(\pi)}{16(\pi)}$ (which may give an answer of 5.25)	M1M1M1A0
	$\frac{100(\pi)}{16(\pi)}$ (which may give an answer of 6.25)	M1M1M0A0

Question	Answer	Mark	Comments
10(a)	Plots the points (1, 60), (2, 30), (3, 20) and (4, 15)	M1	± $\frac{1}{2}$ small square
	Correct smooth curve through correct four points	A1	± $\frac{1}{2}$ small square
	Additional Guidance		
	Ignore any calculations and mark the graph only		
	Points cannot be implied by a bar chart or vertical line graph, but condone crosses at the top of a vertical line graph for M1 and the correct curve superimposed for M1A1		
	For M1, ignore the curve outside the domain $1 \leqslant t \leqslant 4$ For A1, whether or not the curve extends outside the domain $1 \leqslant t \leqslant 4$ it must not have a positive gradient at any point		
	If there is no curve, for M1 there must be no other points with x-coordinate 1, 2, 3 or 4		
	The curve should be a single line with no feathering		
	Unless it affects the shape of the curve (in which case A1 cannot be awarded), ignore incorrect evaluations of 60 ÷ a non-integer value eg 60 ÷ 1.5 = …		

Question	Answer	Mark	Comments
10(b)	Vertical line from $3\frac{1}{2}$ minutes to their graph	M1	$\pm\frac{1}{2}$ small square implied by mark at correct place on the graph or on the vertical axis (but not on the horizontal axis) or by correct reading from their graph
	Correct reading from their graph for $t = 3.5$	A1ft	ft their graph $\pm\frac{1}{2}$ small square
	Additional Guidance		
	Correct reading for their graph, with or without evidence of using graph		M1A1
	No graph in (a)		M0A0
	To score any marks, their graph must be decreasing in the domain $1 \leqslant t \leqslant 4$, but may be a straight line or series of connected straight lines		
	Answer from $60 \div 3.5$ with no graph, or which does not match graph		M0A0
	Reading from 3.3		M0A0

Question	Answer	Mark	Comments
11	**Alternative method 1**		
	330 ÷ (7 + 4) or 30	M1	oe
	7 × their 30 or 210 and 4 × their 30 or 120	M1dep	oe
	45	A1	
	Alternative method 2		
	330 ÷ (7 + 4) or 30	M1	oe
	(7 – 4) × their 30 or 90	M1dep	oe
	45	A1	
	Alternative method 3		
	330 ÷ (7 + 4) or 30	M1	oe
	7 × their 30 or 210 or 4 × their 30 or 120 and 330 ÷ 2 or 165	M1dep	oe
	45	A1	
	Alternative method 4		
	330 ÷ (7 + 4) or 30	M1	oe
	their 30 × 1.5	M1dep	oe
	45	A1	
	Additional Guidance		

Question	Answer	Mark	Comments
12	–9 2 –7 –5 –12	B1	
13	One of (102 →) 100 (8.14 →) 8	M1	
	their 100 = 0.5 × x^2 × their 8 or (x^2 =) their 100 ÷ 8 × 2 or (x^2 =) 100 ÷ their 8 × 2 or 25 or their 8 × 5 × 5 × 0.5 = 100 or 8 × 5 × 5 × 0.5 = their 100	M1dep	oe must have used at least one correct 1 sf value
	5 with M2 seen	A1	
	Additional Guidance		
	If working is done with approximations and with the given values ignore the working with the given values and mark the working with approximations		

Question	Answer	Mark	Comments
14	**Alternative method 1: work out the value of both angles**		
	(b =) 90 ÷ 5 × 3 or 54	M1	oe may be on diagram for b or x
	(x =) $\frac{360 - 90 - \text{their } 54}{3+1}$ or $\frac{216}{4}$	M1dep	oe
	(b =) 54 and (x =) 54 with M2 awarded	A1	
	Alternative method 2: assumes both angles are equal and uses sum of angles in a quadrilateral		
	(b =) 90 ÷ 5 × 3 or 54	M1	oe may be on diagram for b or x
	90 + their 54 + their 54 + 3 × their 54 or 360 – 90 – their 54 – their 54 and either 3 × their 54 or their 162 ÷ 3 or their 162 ÷ 54	M1dep	oe addition of the four angles in the quadrilateral or subtraction of 90 and the two equal angles from 360 and multiplication to work out the fourth angle or division of the fourth angle by 3 or 54 to act as a check
	90 + 54 + 54 + 162 = 360 and 54 × 3 = 162 or 360 – 90 – 54 – 54 = 162 and 162 ÷ 3 = 54 or 162 ÷ 54 = 3	A1	oe
	Alternative method 3: assumes both angles are equal and uses ratio to check 90°		
	5 : 3 : 3 : 9	M1	
	360 ÷ (5 + 3 + 3 + 9) × 5 or 360 ÷ 20 × 5	M1dep	oe
	360 ÷ 20 × 5 = 90 with M2 awarded	A1	
	Additional Guidance		
	Any correct method to work out 54 scores M1 on alt 1 or alt 2		

Question	Answer	Mark	Comments
15(a)	20 48 88 108 120	B1	

15(b)	All 5 points plotted using upper class bounds and their cf values	M1	± $\frac{1}{2}$ small square must be increasing
	Smooth curve or polygon for their cf values	A1ft	± $\frac{1}{2}$ small square must be increasing
	Additional Guidance		
	If (a) is correct, points should be at (10, 20), (20, 48), (30, 88), (40, 108) and (50, 120)		
	For A1, the graph should start at (0, 0) or (1, 0) or (10, 20)		
	For A1, the graph should end at m = 50 unless it followed by a horizontal line adjoining (50, 120)		
	Histogram only		M0A0
	Histogram and graph		Mark curve

15(c)	Line from 15 marks to their graph	M1	± $\frac{1}{2}$ small square implied by mark at correct place on the graph or on the vertical axis (but not on the horizontal axis) or by correct reading from their graph
	Correct reading from their graph for 15 marks	A1ft	± $\frac{1}{2}$ small square
	Additional Guidance		
	Correct reading for their graph, with or without evidence of using graph		M1A1
	No graph in (b)		M0A0
	For M1 and A1ft the domain of their graph must be at least $10 \leqslant m \leqslant 20$ and their graph must be increasing in the domain $10 \leqslant m \leqslant 50$ or from m = 10 if their graph does not extend to m = 50		

Question	Answer	Mark	Comments
16	Correct factorisation of numerator $2(2x - 4x^2)$ or $4(x - 2x^2)$ or $x(4 - 8x)$ or $2x(2 - 4x)$ or $4x(1 - 2x)$ or correct factorisation of denominator $2(6x - 3)$ or $3(4x - 2)$ or $6(2x - 1)$ or correct cancelling by 2 throughout $\frac{2x - 4x^2}{6x - 3}$	M1	oe with negative coefficients
	Correct fraction with numerator $4x(1 - 2x)$ or $-4x(2x - 1)$ and denominator $6(2x - 1)$ or $-6(1 - 2x)$ or $-\frac{4x}{6}$ or $\frac{-4x}{6}$ or $\frac{4x}{-6}$ or $\frac{2x(2 - 4x)}{-3(2 - 4x)}$ or $\frac{2x(2 - 4x)}{3(4x - 2)}$	M1dep	oe with cancelling of 2 throughout eg $\frac{2x(1-2x)}{3(2x-1)}$ or $\frac{2x(1-2x)}{-3(1-2x)}$
	$-\frac{2x}{3}$ or $-\frac{2}{3}x$	A1	allow $\frac{-2x}{3}$ or $\frac{2x}{-3}$
	Additional Guidance		
	Allow multiplication signs up to M1M1		
	Allow $-0.\dot{6}$ for $-\frac{2}{3}$		
	Do not allow $-0.66\ldots$ for $-\frac{2}{3}$		
	For the first M1 only, allow any correct factorisation seen within multiple attempts		

Question	Answer	Mark	Comments
17(a)	$y^2 = \frac{1}{2}y(y + 3)$	B2	oe equation eg $2y^2 = y^2 + 3y$ or $y^2 = 3y$ or $y = 0$ or $y = 3$ or $y = 0$ or 3 B1 $\frac{1}{2}y(y + 3)$ oe expression or an otherwise correct equation using a different unknown or combination of unknowns
	Additional Guidance		
	Allow multiplication signs eg $y \times y = \frac{y}{2} \times (y + 3)$		B2
	$y^2 = \frac{1}{2}y(y + 3)$ followed by incorrect simplification or attempt to solve the equation		B2
	$y^2 = \frac{1}{2}y + y + 3$		B0
	3 only or 0 only or 0 and 3 only		B0
	Do not allow missing or partially missing brackets unless recovered eg1 $y^2 = \frac{1}{2}y \times y + 3$ without correct equation seen eg2 $y^2 = \frac{1}{2}y(y + 3$ without correct equation seen		B0 B0

Question	Answer	Mark	Comments
17(b)	Correct comment or shows correct working	B1	eg1 he hasn't square rooted (correctly) eg2 it should be $\sqrt{8}\,x = 3$ eg3 he should have divided (by 8) before square rooting
	Additional Guidance		
	$\sqrt{8}$ may be given as $2\sqrt{2}$		
	Comment that he shouldn't have a negative answer		B0
	Mathematically incorrect statement		B0
	Correct comment and an incorrect comment		B0
	Example responses		
	He has taken it as $(8x)^2$		B1
	He has divided $8x^2$ by x (instead of square rooting) and square rooted the 9		B1
	He $\sqrt{}$ first when supposed to divide it by 8		B1
	He didn't divide 9 by 8 to get x^2		B1
	At the start he took the 8 over when you want $\sqrt{\frac{9}{8}}$		B1
	Toby should have got $\pm\sqrt{\frac{9}{8}}$		B1
	He should have divided by 8		B0
	Toby didn't square root $8x$		B0
	He hasn't square rooted the $8x^2$ to leave x on its own		B0
	He hasn't square rooted the other side to just get x		B0
	Didn't divide by 8		B0
	He should have divided by $8x$		B0
	He found the square root of 9 but didn't write $\sqrt{8x} = 9$		B0

Question	Answer	Mark	Comments
18(a)	(193 + 7)(193 – 7) or (200)(186) or 200 (×) 186	M1	either order
	(200)(186) = 37 200 or 200 (×) 186 = 37 200	A1	
	Additional Guidance		
	37 200 with correct method not seen		M0A0
	37 200 from 37 249 – 49 only		M0A0
	37 200 from (200)(186) or 200 (×) 186 and 37 249 – 49 also given		M1A1
	Do not award M1 for a 'misread' eg (193 + 2)(193 – 2)		M0A0
18(b)	$(10a + 9b)(10a – 9b)$ or $(9b + 10a)(10a – 9b)$	B1	either order
	Additional Guidance		
	Condone missing final bracket, eg $(10a + 9b)(10a – 9b$		B1
	Condone a multiplication sign eg $(10a + 9b) \times (10a – 9b)$		B1
19	$\frac{1}{9}$	B1	

Question	Answer	Mark	Comments
20(a)	**Alternative method 1: shows that *BAC* = *ACD* and alternate angles**		
	ACD = *ABC*	M1	accept both with same letter on diagram
	ABC = *BAC*	M1	accept both with same letter on diagram
	BAC = *ACD* and alternate segment (theorem) with M2 awarded	M1dep	dep on M2
	Other two correct reasons given with M3 awarded	A1	eg (base angles of) isosceles triangle and alternate angles
	Alternative method 2: shows that *ABC* + *BCD* = 180 and co-interior angles		
	ACD = *ABC*	M1	accept both with same letter on diagram
	ABC = *BAC*	M1	accept both with same letter on diagram
	BCD = 180 – (*BAC* + *ABC*) + *ACD* and *ABC* + *BCD* = 180 and alternate segment (theorem) with M2 awarded	M1dep	oe dep on M2
	Other two correct reasons given with M3 awarded	A1	eg (base angles of) isosceles triangle and (co-)interior angles or allied angles
	The mark scheme for question 20(a) continues on the next page		

Question	Answer	Mark	Comments
20(a) (cont)	**Alternative method 3: line from midpoint of *AB* to *C* is perpendicular to *AB* and *CD***		
	Let *M* be the midpoint of *AB* and *MC* is perpendicular to *AB*	M1	any letter
	MC is perpendicular to *CD*	M1	
	AB and *CD* are both perpendicular to *MC* with M2 awarded	M1dep	oe dep on M2
	Three correct reasons given with M3 awarded	A1	eg (perpendicular bisector of) isosceles triangle and *MC* goes through the centre of the circle and tangent is perpendicular to radius
	Additional Guidance		
	Other correct methods can be found by extending one or more of the lines. For example, by extending *BC* it is possible to use corresponding angles as a proof instead of alternating angles. This should be reflected in the reasons required for the last mark		
	In the scheme, *ACD* (for example) means angle *ACD* and not triangle *ACD*		
	Accept equality of angles indicated by labelling with the same letter, but not by arcs		
	Accept (angle) *B* for angle *ABC* Do not accept (angle) *A* for angle *BAC* or (angle) *C* for angle *ACB* unless intention is clear from annotation of the diagram		
	For the third mark in alternative method 2, accept algebraic expressions for angles if clearly marked on the diagram		
	Do not award marks for an argument based only on assumed values of angles, but ignore 60° marked on diagram, which is for (b)		
	Ignore an angle marked at *ADC*		
	Ignore incorrect statements that do not affect the proof eg *ACD* is an isosceles triangle (but not used in proof)		

Question	Answer	Mark	Comments
20(b)	✓ *AB* is parallel to *DC* ✓ *AC* bisects angle *BCD* *AC* bisects angle *BAD*	B1	
	Additional Guidance		

Question	Answer	Mark	Comments
21	**Alternative method 1: substitution of $2x + p$ for y**		
	$2x + 3(2x + p) = 5p$	M1	oe equation eg $2x + 6x + 3p = 5p$
	$6x + 2x = 5p - 3p$ or $8x = 2p$	M1dep	oe equation with terms collected condone incorrect expansion before rearrangement
	Correct simplified terms $(x =) \frac{p}{4}$ or $\frac{1}{4}p$ or $0.25p$ and $(y =) \frac{3p}{2}$ or $\frac{3}{2}p$ or $1\frac{1}{2}p$ or $1.5p$	A2	A1 one correct simplified term or otherwise correct terms for both with 'p' omitted eg $x = 0.25$ and $y = 1.5$ or correct unsimplified terms for both eg $x = \frac{2p}{8}$ and $y = \frac{6p}{4}$
	Alternative method 2: substitution of $y - p$ for $2x$		
	$y - p + 3y = 5p$	M1	oe equation
	$y + 3y = 5p + p$ or $4y = 6p$	M1dep	oe equation with terms collected
	Correct simplified terms $(x =) \frac{p}{4}$ or $\frac{1}{4}p$ or $0.25p$ and $(y =) \frac{3p}{2}$ or $\frac{3}{2}p$ or $1\frac{1}{2}p$ or $1.5p$	A2	A1 one correct simplified term or otherwise correct terms for both with 'p' omitted eg $x = 0.25$ and $y = 1.5$ or correct unsimplified terms for both eg $x = \frac{2p}{8}$ and $y = \frac{6p}{4}$
	The mark scheme for question 21 continues on the next page		

Question	Answer	Mark	Comments
21 (cont)	**Alternative method 3: elimination of x**		
	$y - 2x = p$	M1	oe with multiplication of both equations
	$y + 3y = 5p + p$ or $4y = 6p$	M1dep	oe addition must be seen if result is incorrect
	Correct simplified terms $(x =) \frac{p}{4}$ or $\frac{1}{4}p$ or $0.25p$ and $(y =) \frac{3p}{2}$ or $\frac{3}{2}p$ or $1\frac{1}{2}p$ or $1.5p$	A2	A1 one correct simplified term or otherwise correct terms for both with 'p' omitted eg $x = 0.25$ and $y = 1.5$ or correct unsimplified terms for both eg $x = \frac{2p}{8}$ and $y = \frac{6p}{4}$
	Alternative method 4: elimination of y		
	$3y - 6x = 3p$	M1	oe with multiplication of both equations
	$2x - (-6x) = 5p - 3p$ or $8x = 2p$	M1dep	oe subtraction must be seen if result is incorrect
	Correct simplified terms $(x =) \frac{p}{4}$ or $\frac{1}{4}p$ or $0.25p$ and $(y =) \frac{3p}{2}$ or $\frac{3}{2}p$ or $1\frac{1}{2}p$ or $1.5p$	A2	A1 one correct simplified term or otherwise correct terms for both with 'p' omitted eg $x = 0.25$ and $y = 1.5$ or correct unsimplified terms for both eg $x = \frac{2p}{8}$ and $y = \frac{6p}{4}$

Question	Answer	Mark	Comments
22(a)	– 3**b** + 6**a** + 7.5**b** (= 6**a** + 4.5**b**) or 6**a** + 7.5**b** – 3**b** (= 6**a** + 4.5**b**) or 6**a** + 7.5**b** – (6**a** + 4.5**b**) = 3**b**	B1	oe rearranged equation using all 5 terms
	Additional Guidance		
	3**b** + 6**a** + 4.5**b** = 6**a** + 7.5**b**	B1	
	6**a** + 4.5**b** + 3**b** = 6**a** + 7.5**b**	B1	
	7.5**b** – 3**b** = 4.5**b**, so 6**a** + 4.5**b**	B0	
	6**a** + 7.5**b** – 3**b** = 4.5**b**	B0	

Question	Answer	Mark	Comments
22(b)	**Alternative method 1: equal ratios from *k*a + 3b and 6a + 4.5b**		
	(*BC* =) k**a** + 3**b** or $k : 6 = 3 : 4.5$ or $k : 3 = 6 : 4.5$	M1	oe ratio
	$3 \times 6 \div 4.5$ or 4**a** + 3**b**	M1dep	oe
	4	A1	
	Alternative method 2: scale factor from *k*a + 3b and 6a + 4.5b		
	(*BC* =) k**a** + 3**b** or $4.5 \div 3$ or $\frac{3}{2}$ or $3 \div 4.5$ or $\frac{2}{3}$ or $4.5 \div 6$ or $\frac{3}{4}$ or $6 \div 4.5$ or $\frac{4}{3}$	M1	oe fractions or decimals
	$6 \div$ their $\frac{3}{2}$ or $6 \times$ their $\frac{2}{3}$ or $3 \div$ their $\frac{3}{4}$ or $3 \times$ their $\frac{4}{3}$ or 4**a** + 3**b**	M1dep	oe
	4	A1	
	The mark scheme for question 22(b) continues on the next page		

Question	Answer	Mark	Comments
22(b) (cont)	**Alternative method 3: equal ratios from (k + 6)a + 7.5b and 6a + 4.5b**		
	(BD =) k**a** + 6**a** + 7.5**b** or (BD =) (k + 6)**a** + 7.5**b** or (k + 6) : 6 = 7.5 : 4.5 or (k + 6) : 7.5 = 6 : 4.5	M1	oe ratio
	6 × 7.5 ÷ 4.5 – 6 or 4**a** + 3**b**	M1dep	oe
	4	A1	
	Alternative method 4: scale factor from (k + 6)a + 7.5b and 6a + 4.5b		
	(BD =) k**a** + 6**a** + 7.5**b** or (BD =) (k + 6)**a** + 7.5**b** or 7.5 ÷ 4.5 or $\frac{5}{3}$ or 4.5 ÷ 7.5 or $\frac{3}{5}$ or 4.5 ÷ 6 or $\frac{3}{4}$ or 6 ÷ 4.5 or $\frac{4}{3}$	M1	oe fractions or decimals
	6 × their $\frac{5}{3}$ – 6 or 6 ÷ their $\frac{3}{5}$ – 6 or 7.5 ÷ their $\frac{3}{4}$ – 6 or 7.5 × their $\frac{4}{3}$ – 6 or 4**a** + 3**b**	M1dep	oe
	4	A1	
	Additional Guidance for question 22(b) is on the next page		

Question	Answer	Mark	Comments

	Additional Guidance	
22(b) (cont)	Check the diagram for working	
	If working is not seen, only accept exact decimal values in place of fractions for method marks	
	Answer 4 with no working or no incorrect working	M1M1A1
	Assumes that *BC* is $3a + 2.25b$ (half the length of *CD*) or that *BC* is $2a + 1.5b$ (one third of the length of *CD*)	M0M0A0 M0M0A0
	4a on the answer line does not get the A mark, but may have scored the method marks	

Question	Answer	Mark	Comments
23	**Alternative method 1**		
	$(8^4 =)\ (2^3)^4$ or 2^{12} or $(32^{\frac{2}{5}} =)\ (2^5)^{\frac{2}{5}}$ or 2^2	M1	
	2^{12} and 2^2	M1dep	or calculation in the form $2^a \div 2^b$ where $a - b = 10$ or $2^c \times 2^d$ where $c + d = 10$
	2^{10}	A1	Accept $m = 10$
	Alternative method 2		
	$(8^4 =)$ 4096 or $(32^{\frac{2}{5}} =)$ 4	M1	
	1024	M1dep	
	2^{10}	A1	Accept $m = 10$
	Additional Guidance		
	Note that 1024 from 32 × 32 scores 2 marks if 1024 is their final numerical answer However, if they then try to find $\sqrt[5]{1024}$ they are clearly processing $(32^{\frac{2}{5}} =)$, so this would only score 0 marks without further work		
	If a numerical method and an index method are both attempted and an incorrect answer is given, award up to M1M1 from the better method		
24	–1	B1	

Question	Answer	Mark	Comments
25(a)	(gradient of OP =) $\frac{8-0}{4-0}$	M1	oe eg (gradient of OP =) $\frac{8}{4}$
	(gradient of OP =) 2 or $\frac{2}{1}$ and $-1 \div 2 = -\frac{1}{2}$ or $2 \times -\frac{1}{2} = -1$ with M1 seen	A1	oe accept 'negative reciprocal, so $-\frac{1}{2}$, or 'product of gradients is –1, so $-\frac{1}{2}$, oe comment
	Additional Guidance		
	$4 \div 8 = \frac{1}{2}$ but slope is negative, so $-\frac{1}{2}$		M0A0
	Do not accept a gradient including x eg $\frac{8}{4} = 2$, so gradient of $OP = 2x$, product of gradients is –1, so $-\frac{1}{2}x$		M1A0

Question	Answer	Mark	Comments
25(b)	**Alternative method 1: $y = -\frac{1}{2}x + c$ and substitutes 8 and 4**		
	$8 = -\frac{1}{2} \times 4 + c$ or $(c =)$ 10	M1	oe implied by $y = -\frac{1}{2}x + 10$
	$0 = -\frac{1}{2}x$ + their 10 or $(x =)$ 20	M1dep	oe
	their 20^2 + their 10^2 or 500 or $\sqrt{500}$	M1dep	oe eg $2\sqrt{125}$ dep on M2
	$10\sqrt{5}$	A1	accept $a = 10$ with $\sqrt{500}$ seen
	Alternative method 2: uses the formula for a line and substitutes $x = 0$ and $y = 0$		
	$y - 8 = -\frac{1}{2}(x - 4)$ and substitutes $x = 0$ or $y = 0$ or $(x =)$ 20 or $(y =)$ 10	M1	oe equation eg $x + 2y = 20$
	$y - 8 = -\frac{1}{2}(x - 4)$ and substitutes $x = 0$ and substitutes $y = 0$ or $(x =)$ 20 and $(y =)$ 10	M1	oe equation eg $x + 2y = 20$
	their 20^2 + their 10^2 or 500 or $\sqrt{500}$	M1dep	oe eg $2\sqrt{125}$ dep on M2
	$10\sqrt{5}$	A1	accept $a = 10$ with $\sqrt{500}$ seen
	The mark scheme for question 25(b) continues on the next page		

Question	Answer	Mark	Comments

25(b) (cont)	**Alternative method 3: uses formula for gradient with points *A* and *B***		
	$\frac{8-0}{4-x} = -\frac{1}{2}$ or $(x =)\ 20$	M1	oe correct method to work out the x-coordinate of point A
	$\frac{y-8}{0-4} = -\frac{1}{2}$ or $(y =)\ 10$	M1	oe correct method to work out the y-coordinate of point B
	their 20^2 + their 10^2 or 500 or $\sqrt{500}$	M1dep	oe eg $2\sqrt{125}$ dep on M2
	$10\sqrt{5}$	A1	accept $a = 10$ with $\sqrt{500}$ seen
	Additional Guidance		
	Check the diagram and 25(a) for possible correct working or values eg 1 20 marked on axis at A eg 2 10 marked on axis at *B*		M1 M1
	On alternative method 2, if using $y - 8 = -\frac{1}{2}(x - 4)$, they must substitute $x = 0$ or $y = 0$ for M1 and both separately for M1M1		
	On alternative method 2, incorrect rearrangement of $y - 8 = -\frac{1}{2}(x - 4)$ can score up to 3 marks eg $y - 8 = -\frac{1}{2}(x - 4)$, $2y - 8 = -x - 4$, when $y = 0$, $x = 4$, when $x = 0$, $y = 2$, $\sqrt{4^2 + 2^2} = \sqrt{20}$		M1M1M1

26	$(x - -2)^2$ or $(x + 2)^2$ or $a = 2$	M1	oe implied by $x^2 + 2x + 2x + 4$ (+ b) or $x^2 + 4x + 4$ (+ b)
	$1 = (3 + 2)^2 + b$	M1dep	oe
	–24	A1	accept (–2, –24)
	Additional Guidance		
	$(x - 2)^2$ $1 = (3 - 2)^2 + b$		M0 M0

Question	Answer	Mark	Comments
27	$\sin 60° = \frac{\sqrt{3}}{2}$ or $\tan 30° = \frac{\sqrt{3}}{3}$ or $\frac{1}{\sqrt{3}}$ or $\tan 30° \left(= \frac{\sin 30}{\cos 30}\right) = \frac{1/2}{\sqrt{3}/2}$	M1	oe may be in a table may be implied by position in multiplication
	$\frac{\sqrt{3}}{2} \times \frac{1}{\sqrt{3}} = \frac{1}{2}$ or $\cos x = \frac{1}{2}$ or $(x =) \cos^{-1} \frac{1}{2}$	M1dep	oe works out the value of cos x as a fraction or decimal with no surd values
	60 with M2 awarded	A1	
	Additional Guidance		
	cos x = 60 does not score the final mark		